

MATHEMATICAL CHALLENGE 2016–2017

Entries must be the unaided efforts of individual pupils.

Solutions must include explanations and answers without explanation will be given no credit.

Do not feel that you must hand in answers to all the questions.

CURRENT AND RECENT SPONSORS OF MATHEMATICAL CHALLENGE ARE

The Edinburgh Mathematical Society, The Maxwell Foundation, Professor L E Fraenkel,

The London Mathematical Society and The Scottish International Education Trust.

The Scottish Mathematical Council is indebted to the above for their generous support and gratefully acknowledges financial and other assistance from schools, universities and education authorities.

Particular thanks are due to the Universities of Aberdeen, Edinburgh, Glasgow, Heriot Watt, St Andrews, Stirling, Strathclyde and to Bearsden Academy, Kelvinside Academy and Northfield Academy.

Senior Division: Problems 1

- S1. A pyramid stands on horizontal ground. Its base is an equilateral triangle with sides of length a , the other three edges of the pyramid are of length b and its volume is V . Show that the volume of the pyramid is

$$V = \frac{1}{12}a^2\sqrt{3b^2 - a^2}.$$

The pyramid is then placed so that a non-equilateral face lies on the ground. Find the height of the pyramid in this position.

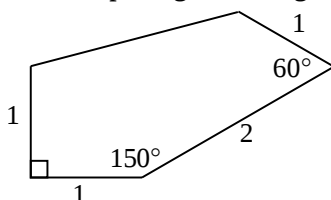
- S2. Eight islands each have one or more air services. An air service consists of flights to and from another island, and no two services link the same pair of islands. There are 17 air services in all between the islands.

Show that it must be possible to use these air services to fly between any pair of islands.

- S3. How many distinct solutions consisting of positive integers does this system of equations have?

$$\begin{array}{rclcl} x_1 + x_2 + x_3 & & & & = 5 \\ & y_1 + y_2 + y_3 & & & = 5 \\ & & z_1 + z_2 + z_3 & = 5 \\ x_1 + & & y_1 + & & z_1 & = 5 \\ & x_2 + & & y_2 + & & z_2 & = 5 \\ & & x_3 + & & y_3 + & & z_3 & = 5 \end{array}$$

- S4. Over many centuries, tilings have been important to mathematicians and to society. To use a single tiling to cover a plane is very useful. For a few cases, regular polygons (equilateral triangles, squares, regular hexagons) can be used but in order to do that, their angle needs to be a factor of 360° . It is not possible to have a tiling of regular pentagons. However, non-regular pentagons are known which will tile the plane. There was much excitement in August 2015 when a team in the United States discovered a hitherto unknown pentagonal tiling and it is shown in the diagram.



Use the lengths given and angle sizes to calculate its exact area.

SEE OVER FOR QUESTION S5.



Mathematical Challenge Problems 1

SENIOR DIVISION 2016-2017

PLEASE USE CAPITALS TO COMPLETE

SURNAME

OTHER NAME(S)
(underline the one
you prefer)

SCHOOL

AGE

YEAR OF STUDY

FOR OFFICIAL USE

Marker

Marks

1	2	3	4	5

Total

— — — — - C U T A L O N G H E R E — — — —

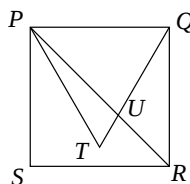
Please write your solutions on A4 paper and staple the above form to them.

PLEASE WRITE YOUR NAME ON EVERY PAGE.

Send your entry through your school to the section organisers.

For further information on the competition, please see the Information Circular, which has been distributed to all secondary schools. Please contact the local organiser, whose name and address are given above, if you require a further copy.

S5.



The diagram shows square $PQRS$ with sides of length 1 unit. Triangle PQT is equilateral. Show that the area of triangle UQR is $(\sqrt{3} - 1)/4$ square units.

END OF PROBLEM SET 1

CLOSING DATE FOR RECEIPT OF SOLUTIONS :

27th September 2016

Look out for Problems 2 in early December!

Look on the SMC web site:

www.scot-maths.co.uk

for information about Mathematical Challenge